

DYNAMIC SIMULATION AS A TOOL FOR OPTIMIZING PRESSURE CONTROL VALVE PERFORMANCE

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ABSTRACT

Control valves installed for pump station pressure control are typically tuned and commissioned at the low end of the flow range and well below the safe operating limits in order to avoid pressure excursions and line shutdowns during commissioning.

Tuning parameters selected for best performance at low flowrates often produce poor performance at high flowrates requiring dampened tuning parameters and slower valve actuator speeds. This results in sluggish responses to pressure changes.

Enbridge has undertaken a case study to examine three control valves known to be problematic. The goal of this study was to produce an optimal tuning strategy that could be implemented with a high degree of confidence over the entire range of operating conditions. To accomplish this, the IDEAS (AMEC Technologies, Inc.) dynamic simulation software package was utilized.

The pipeline was modeled from the pump station upstream of the station of interest to the downstream pump station. The model consists of pipeline sections, pumps, control valves and other process elements that are hydraulically linked. Station discharge and suction pressures are controlled via PI controllers with adjustable set points, ramp rates and tuning constants. Valve full stroke actuator speed can also be varied.

Information required to develop the simulation model included station elevations, pipeline lengths, pump curves and control valve C_v curves. The three simulation models developed for this study have been calibrated against process data by adjusting piping resistances.

The inherent nonlinearities present in the control valve

system were quantified through use of the simulation model. Various strategies to alleviate the adverse effects of these nonlinearities have been studied. Use of a simulation tool also resulted in increasing the awareness of trade-offs present in design and tuning of control valve systems.

KEYWORDS

Dynamic Simulation, Pressure Control Valve, Lambda Tuning, Output Linearization

INTRODUCTION

Enbridge operates a complex and lengthy system of pipelines transporting a range of liquids including crude oils, refined products and Natural Gas Liquids (NGL). Pressure control valves (PCVs) are used as the primary control element at the majority of pump stations and as a secondary control element at variable frequency drive controlled pump stations.

Enbridge's control valve selection practices have historically departed from industry norms and accordingly are viewed as "oversized". This, coupled with applying control strategies for low-end flowrates, have resulted in conservative or dampened PCV performance.

Enbridge has undertaken to re-examine PCV selection and control strategies.

Operations identified three pump station PCVs and associated control systems known to have operating difficulties. These stations were also thought to be a representative sample of a larger number of similar installations.

A plan was developed and executed in five stages:

1. Record field data and perform off-line valve tests (this work was performed by Mr. Larry Neumeister of Spartan Controls).
2. Develop and calibrate dynamic simulation models.
3. Execute numerous model scenarios to generate data for existing conditions.
4. Develop and test alternative tuning methodologies using the models.
5. Implement new tunings at subject stations.

This paper will focus on dynamic simulation models and their efficacy in engineering optimal tuning of control valves.

NOMENCLATURE

A	Pipe cross sectional area
C_v	Valve flow coefficient
D	Pipe diameter
F	Friction losses
f	Fanning friction factor
I	Controller integral reset time
K_c	Controller proportional gain
L	Pipe length
P	Pressure
PCV	Pressure Control Valve
T_{86}	Time at which controller response reaches 86.5% of the step change
v	Fluid velocity
λ	Closed-loop time constant
η	Dynamic viscosity
ρ	Density

SIMULATION MODEL DESCRIPTION

Several technical requirements for the simulation models were deemed important prior to their development:

1. Hydraulic calculations of line and valve losses, pump head generated, etc. must be performed.
2. Detailed modeling of the pressure control valve system must be possible.
3. The models must be able to execute fast enough for many runs to be performed in any given study.
4. The models must be easily (re-)configurable so instrumentation/engineering staff can perform “what-if” analyses.

The IDEAS dynamic simulation package (AMEC Technologies Inc.) was identified as fulfilling these requirements. IDEAS includes libraries of process components that are connected to form a dynamic model worksheet having

the appearance of a process flowsheet. One of the three models developed for this project is shown in Figure 1. The layout of all three models is very similar, but each must be configured using process information specific to each site (pump curves and valve C_v curves, for example). Equipment specific information is entered through object dialog boxes. Simulation results are obtained in IDEAS plotter objects or can be exported through dynamic data exchange (DDE) to spreadsheet programs. Snapshots of steady-state process conditions can be saved. The model can then be restarted at a later time from these saved process conditions.

As is seen in Figure 1, the scope of the model includes the Study Station (“Kingman Line 3” in the figure), as well as the stations upstream and downstream of the Study Station. The upstream and downstream stations are modeled as fixed discharge and suction pressures respectively that can be changed by the user based on the number of operating pumps. Limiting the model to these three stations reduces the amount of information (and time) required to build the models while increasing their execution speed. Experience to date indicates this configuration provides sufficient accuracy to characterize controller dynamics at the Study Station.

The hydraulic pressure/flow solution algorithm employed in the IDEAS simulation program assumes pseudo steady-state behaviour of the piping network. That is, piping network dynamics are very fast relative to ‘true’ dynamics in tanks, PID controllers, pump start-ups or motor operated valves. The algorithm solves a steady-state form of Bernoulli’s equation at each time step until the mass balance at all nodes is within a user specified tolerance. The user can adjust the number of iterations per time step or the size of the time step to achieve either greater speed or accuracy in the hydraulic calculations.

Since the simulator methodology is based on mass balancing at nodes, it is not feasible to model multiple products in the pipeline (i.e. batched operations). For the purposes of determining tuning strategies for pressure control valve systems, this limitation has not been problematic. Compensating for calibration data collected during periods where multiple products exist in the pipeline is discussed in the “Model Calibration” section of this paper.

CONTROL VALVE SYSTEM MODEL

According to the EnTech Control Valve Dynamic Specification [1], a control valve system includes the actuator, drive train, positioner and valve. In this study, specific information about the valve characteristic curve (flow versus % open) and the actuation speed have been included. Details of the mechanical/hydraulic couplings within the valve drive train and positioner are only included in

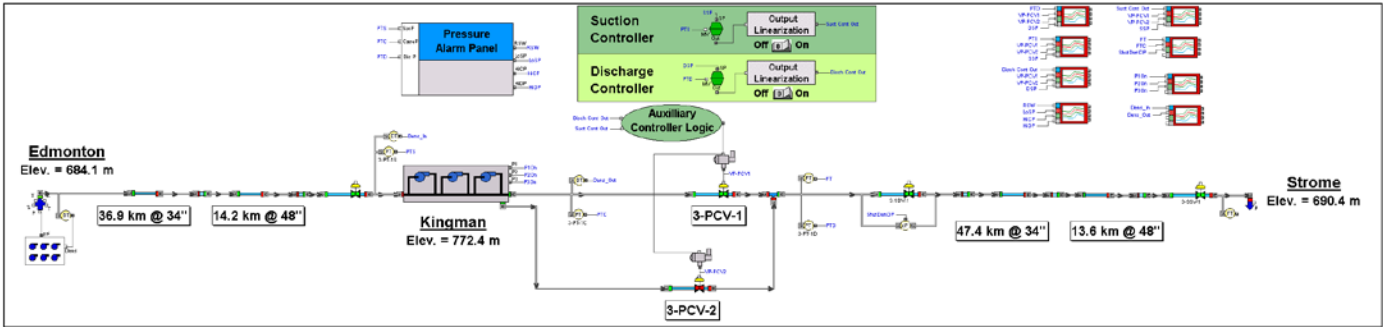


Figure 1 IDEAS Simulation Worksheet for Kingman Line 3

the model through empirically determined control valve step responses.

The pressure control valves at each Study Station are given in Table 1. At the Kingman Line 3 station, the two globe valves are arranged in parallel with 3-PCV-1 traveling over the entire range of controller output while 3-PCV-2 only starts to open when 3-PCV-1 is 50% open.

Station	Valve Description
Kingman Line 3	2 x 20" Acme Flow Globe Valves
Odessa Line 3	1 x 20" Fisher V250 Ball Valve
Glenavon Line 3	1 x 12" Fisher V250 Ball Valve

Table 1. Pressure Control Valves at the PCV Simulation Study Stations

Manufacturer’s information about the characteristic curve for Kingman Station was of poor quality. Therefore, the Cv curve had to be determined from process data, which was collected as part of a control valve audit [2]. A combined Cv curve was first determined for both operating valves. Nameplate information indicated that the valves were identical. Accordingly, the Cv curve for an individual valve was determined by:

1. Using the combined Cv curve over the first 50% of valve travel since only 3-PCV-1 is operating in this range,
2. Adjusting the Cv of each individual valve for 100% controller output in the IDEAS simulation to get the combined Cv determined from process data,
3. Making a linear interpolation from the 50% controller output position to 100% controller output.

The resulting Cv curve for 3-PCV-1 or 3-PCV-2 was found to be approximately linear as shown in Figure 2. This contrasts with nameplate information that indicated quick opening characteristics.

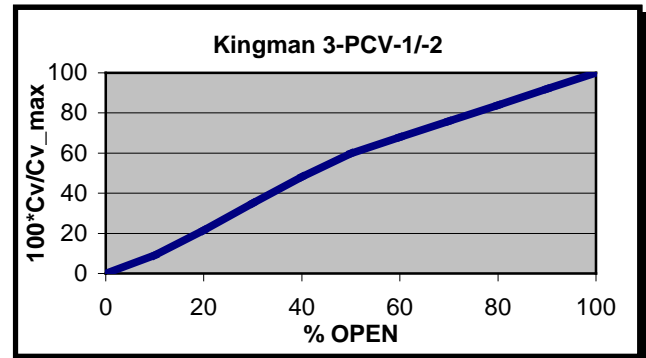


Figure 2. Cv Curve for Kingman 3-PCV-1 and 3-PCV-2

For Odessa and Glenavon stations, up to date manufacturer’s data was available for the control valves. Both valves have equal percentage characteristics as illustrated in Figure 3 below (Odessa station shown).

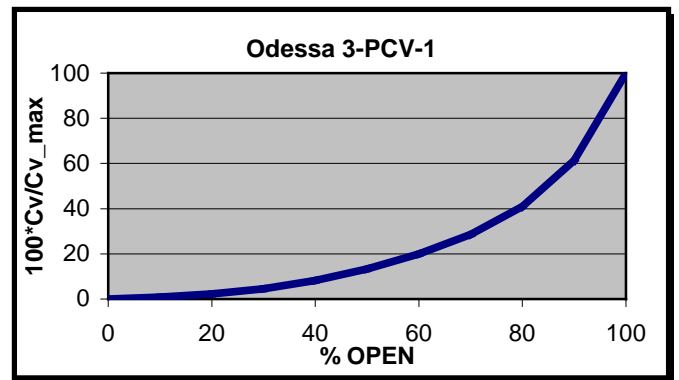


Figure 3. Cv Curve for Odessa 3-PCV-1

Electro-hydraulic valve actuators are present at all three stations. As-installed PCV opening/closing times were determined during the control valve audit [2] by introducing immediate 100-0-100% step inputs to the valve. Speeds during these tests are limited by the valve actuators that influence the rate of response to large or sudden load disturbances. Results are summarized in Table 2 below:

Valve	Closing (sec/100%)	Opening (sec/100%)	Dead Time (sec)
Kingman 3-PCV-1	3.4	14.8	0.2
Kingman 3-PCV-2	2.0	16.4	0.2
Odessa 3-PCV-1	11.2	12.0	0.2
Glenavon 1-PCV-1	8.4	9.0	0.0

Table 2 Stroking Speeds for Pressure Control Valves

Tests were also conducted to determine the valve dynamics in response to very small step changes, with the valve starting from rest. This test reveals the (assumed 1st order) valve dynamics typical during regulatory control in response to small disturbances. Results are summarized in Table 3 below:

Valve	Dead Time (sec)	Time Constant (sec)
Kingman 3-PCV-1	2.2	2.9
Kingman 3-PCV-2	2.4	1.6
Odessa 3-PCV-1	0.2	1.5
Glenavon 1-PCV-1	1.1	0.6

Table 3 Dynamics of Pressure Control Valves

The actual valve dynamics during process operation likely fall between the two extremes represented by results in Tables 2 and 3. Since the prime focus of the dynamic simulation study is to ensure good response to large disturbances, two simulation scenarios were implemented based on different assumptions about valve dynamics:

1. The PCV follows actuator stroking speeds (per Table 2). No dead time exists in the valve.
2. The PCV follows actuator stroking speeds (per Table 2). One second dead time exists in the valve.

The first assumption gives the ideal or best case performance for the control valve system. The second assumption will allow examination of how dead time influences control valve system response. The *EnTech Control Valve Dynamic Specification* [1] discusses the destabilizing influence of dead time in control valve systems.

In addition to the control valve itself, components of the discharge and suction pressure controller logic present in the model include:

- Proportional-Integral controllers with adjustable tuning constants
- Optional set-point ramping
- Optional controller output ramping
- Optional controller output clipping
- Controller output linearization capability

MODEL ALARMS

The following alarm limits are programmed into each model:

- Low Suction Pressure
- High Suction Pressure
- High Pump Discharge Pressure
- High PCV Discharge Pressure
- Low Flowrate (according to individual pump specifications)

Animated indicators on the pump station icon or Pressure Alarm Panel (see Figure 1) notify the user when alarm limits are breached. Limits are recorded during simulation runs and can be exported to spreadsheet programs along with other process data.

The simulator is also capable of notifying the user of conditions in the control valve or pump that cause cavitation.

MODEL CALIBRATION

Each of the three models has been calibrated using data collected during the control valve audits [2] as well as from the process data archiving system. Several (8-10) points were identified for each station at which the process operated at steady state. Product information was recorded only once every 24 hours, so pipeline product composition had to be time synchronized to process data at these steady state points.

At any of the steady-state points identified for calibration, the pipeline was filled with several different products of widely varying characteristics. For example, one of the 20" diameter lines contained batches of either a synthetic crude blend (865 kg/m³, 6.4 cP) and NGL (548 kg/m³, 0.2 cP). Over a typical 50 km section of pipeline the calculated pressure drop for each of these two fluids flowing at 1200 m³/hr differed by over 200 psi. This implied that a method accounting for different products in the pipeline during model calibration was required. This was accomplished by adjusting the length of the pipeline sections containing products other than that present at the flowrate measurement point. The equation used for adjustment of the piping lengths during calibration is:

$$\frac{L_1^*}{L_1} = \left(\frac{\rho_1}{\rho_{ref}} \right)^{1.84} \quad (1)$$

where: L_1^* is the modified pipe length
 L_1 is the original pipe length
 ρ_1 is the product density to be modified
 ρ_{Ref} is the product density at the reference measurement point.

A derivation of Eq. (1) is given in Appendix A.

Pipe roughness coefficients, e (in meters), are then adjusted to provide the best fit to process data. Four responses were used to compare model fit – discharge, suction, and case pressures along with flowrate. The model fit plot for measured versus predicted discharge pressure at Kingman Line 3 is shown in Figure 4 and shows excellent fit to the collected process data. Other stations have similarly excellent fit for all process variables measured. Further model validation with a separate dataset has not been conducted.

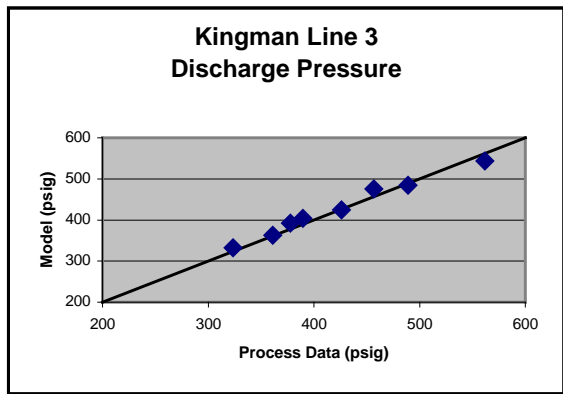


Figure 4 Simulation model fit for discharge pressure at Kingman Line 3 station.

CONTROLLER TUNING OBJECTIVES

There are two primary objectives of a control system: stability and performance. The first objective is that it maintains stability of the process. If so, then control system performance can be judged by the speed at which the process variable reaches its set point. The following measures of controller stability and controller performance are used in this study:

1) *Controller Stability* - The control valve system is deemed to be unstable if it exhibits:

- Oscillations approaching or around a set point
- Overshoot in approaching a set point

Control system stability is judged on a strictly qualitative basis.

2) *Controller Performance* - The primary measure of closed loop controller performance used in this study is T_{86} -- the time in seconds that the controller response takes to reach 86% of its set point value. A smaller T_{86} implies that the control system has faster performance.

The *EnTech Control Valve Dynamic Specification* [1] suggests that the ideal closed-loop control valve system response for most industrial controllers is first-order. Rules have been developed which allow tuning of a controller to achieve a desired closed loop response and are often referred to

as Lambda Tuning. Appendix B outlines a procedure for determining Lambda Tuning constants.

Achieving the desired results from the Lambda Tuning procedure is contingent on system linearity. Hydraulic systems with oversized control valves generally display nonlinear characteristics. Reducing the negative impacts of these nonlinearities on the control system can be accomplished either by modifying the valve trim or through compensation of the controller action. The equal percentage trim shown in Figure 3 is an example of the former while Output Linearization is an example of the latter. It is applied as a modifying function to the output of the controller as illustrated in Figure 5 below [3].

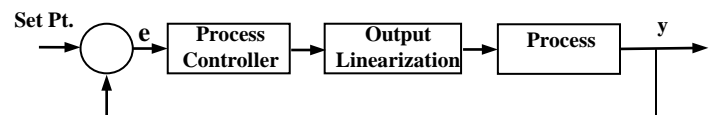


Figure 5 Block diagram illustrating the application of Output Linearization

The basic idea of Output Linearization is to first determine the normalized process response as a function of controller output, and then determine the inverse of this function that will make the closed loop process linear. Appendix B outlines a more detailed procedure for determining the inverse compensating function. Note that once the linearization function has been applied to the system, the controller is re-tuned using the Lambda Tuning procedure.

In determining Lambda tuning constants, open-loop step (or bump) tests must be performed on the process. In practice, operations personnel typically resist implementing additional disturbances. The process simulator offers the potential to determine a preliminary process model from which Lambda tuning constants can be derived without disturbing the actual process. Furthermore, process step tests are typically performed in one operating region, far from potential alarm or shut down limits. Identification of process models and tuning constants using the process simulator can be performed over the entire range of operation without jeopardizing production or safety.

EXAMPLE

Kingman Line 3 station discharge pressure control will be used as an example to show the range of information that the process simulator can provide to instrumentation and control engineers. The aim of the example is to examine how the control system responds to large step disturbances under different controller strategies. A crude oil product is assumed present in the pipeline for this example.

The discharge pressure controller is a Modicon PLC utilizing proportional plus integral logic. In practice, a high select function chooses between suction and discharge pressure

control. This feature has been disabled in the simulator for the purposes of this study.

The gain diagram for the currently installed system at Kingman Line 3 station is shown in Figure 6. Note that the valve is not calibrated 0-100% for 0-100% controller output. Also note that the flowrate relative gain for 3-PCV-2 is not shown as it is effectively zero over the range of [50-100]% controller output.

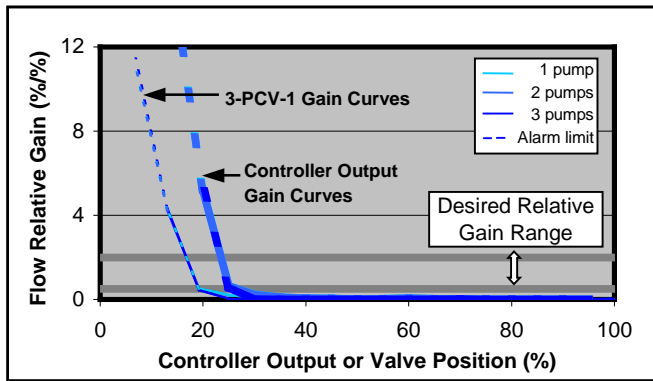


Figure 6 Relative Flowrate Gain Curves for the Kingman Station current control system.

The extreme nonlinearity of this control valve system is evident. The valve exerts essentially zero control over the flow for controller outputs greater than 30% open. The relative gain then rises rapidly until operating limitations are reached. Differences in the relative gain between the lower and upper controller outputs are in the order of 1000:1 making controller tuning for the entire operating range extremely difficult. Indeed, examination of process data reveals that Kingman Line 3 station experiences process instabilities below 25% controller output [2].

Both Lambda Tuning and Output Linearization were compared against current system performance. A value of $\lambda=3$ seconds has been chosen for all cases. Therefore, good controller performance would yield a T_{86} of approximately 6 seconds. One second dead time was assumed to be present in the valve for all cases considered. For Lambda Tuning alone, the process gain and time constant were determined by an open-loop step test performed on the (simulated) process at minimum flow conditions. The process model identified in this region has a high gain that leads to conservative controller tuning constants. The Output Linearization function was determined via the method outlined in Appendix B. The discharge pressure controller was tuned based on the maximum relative gain across the entire feasible operating region. A summary of the proportional-integral controller tuning constants determined for each of the cases is shown in Table 4.

Note that the integration rates are quite fast for both the Lambda Tuning and Output Linearization cases. This results

from the valve assumption that it travel at the valve stroking speed. In practice, there may be limitations on the maximum speed of controller integration due to pre-defined constraints in the PLC program that need to be observed.

Case	K_c (%/%)	I (sec/rep)
Current	2.0	5
Lambda Tuning	0.01	0.15
Output Linearization	0.12	0.15

Table 4 Proportional-Integral tuning constants for Kingman Line 3

The gain curve for the system after application of Output Linearization is shown in Figure 7. The relative flowrate gain curve is now reasonably flat over the entire range of controller output. It also lies within the desired range of [0.5 – 2.0] making stable controller tunings much easier to achieve. Note that the relative gain curve with respect to 3-PCV-1 position is essentially unchanged (except for valve recalibration). This curve would change only if the valve trim or other process modifications have been made. The extremely narrow valve controllable range requires that it have excellent minimum step resolution. A related simulation study showed that modifying valve trim characteristics from linear to equal percentage increased the controllable span from 15% to 34%, thereby relaxing requirements on the minimum valve step resolution.

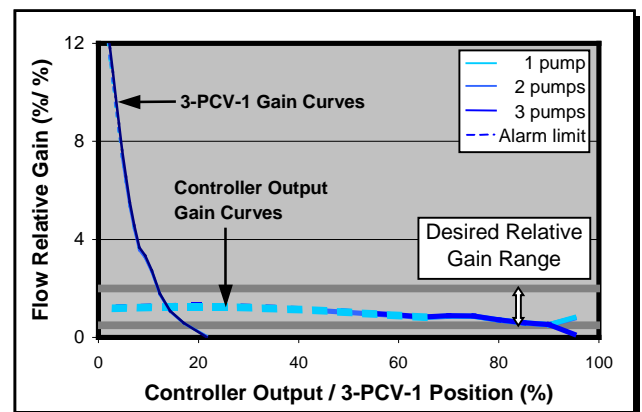


Figure 7 Relative Flowrate Gain Curves for Kingman Station with Output Linearization.

The performance of the control system was tested at both minimum and maximum flowrate conditions by introducing immediate discharge pressure set point changes without ramping. Figure 8 shows results for the tests performed at minimum flowrate conditions. The current control system is unstable at the baseline conditions since the controller output is below 25%, as previously noted. The performance of the Lambda Tuned and Output Linearization systems are similar to one another. This is not surprising since the Lambda Tunings are based on a process model identified at minimum flowrate

conditions. Note that the Output Linearization response is approximately 1st order whereas the Lambda Tuning case is not.

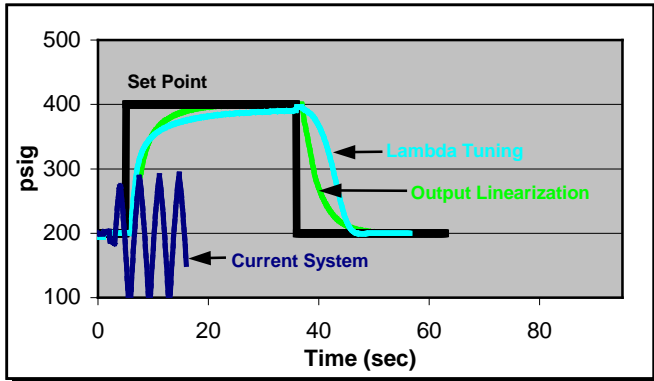


Figure 8 Discharge pressure step responses for Kingman Station at minimum flowrate conditions.

Figure 9 shows discharge pressure responses to 50 psi set point changes from maximum flowrate conditions. The current system is stable at the baseline conditions but induces instabilities as the valve position moves below 25% open. Lambda Tuned controller response is stable but very slow. For the initial step response, a large delay time is experienced as the valve moves through the region of very low process gain seen in Figure 6. Since Output Linearization compensates for this very low process gain, it provides excellent performance at these conditions. Again, note that the discharge pressure response at these conditions is approximately 1st order.

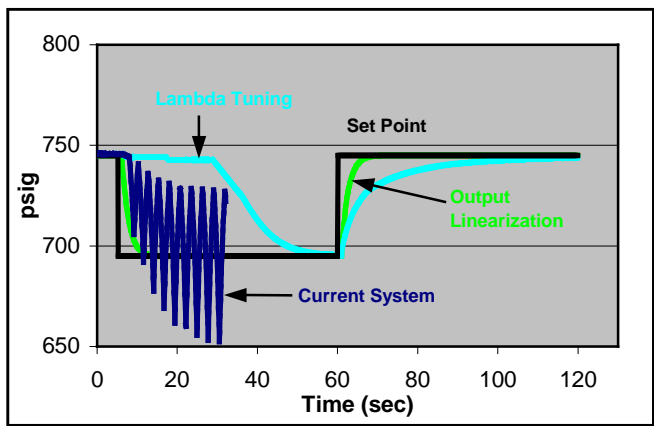


Figure 9 Discharge pressure step responses for Kingman Station at maximum flowrate conditions.

Stability and performance of the three control schemes tested is summarized in Table 5.

Minimum Flow Conditions		
Case	Stable	T ₈₆ (sec)
Current	No	N/A
Lambda Tuning	Yes	9
Output Linearization	Yes	7
Maximum Flow Conditions		
Case	Stable	T ₈₆ (sec)
Current	No	N/A
Lambda Tuning	Yes	20-40
Output Linearization	Yes	4

Table 5 Summary of Kingman Line 3 controller system stability and performance

Application of Output Linearization clearly provides stable and consistent performance over the entire operating range. This is evident in Figures 8 and 9 with reasoning behind the improvements shown by the gain plots of Figures 6 and 7. If implemented, these modifications to the control system would be effective over the entire range of process operation.

CONCLUSIONS

The use of dynamic simulation models has enabled Enbridge to explore multiple factors in determining optimal PCV tunings including:

- responses to step changes over a full range of flowrates
- various actuation speeds
- differing deadtimes
- alternative tuning methods

Another benefit of the simulator is its provision of additional insight compared to a purely empirical approach. For example, the effective control range of each valve was graphically demonstrated which underscored the requirement for superior step resolution of the control element.

Analysis of PCV thresholds for cavitation and choked flow, and assessing control valve replacement alternatives are further applications of simulation being considered. Extending the models to include a series of pump stations would allow examination of the interplay of pressure control amongst stations.

The simulation model used in this study requires detailed information about valve dynamics, which has been obtained from field data. Collection of valve dynamic data involves less risk than performing step tests on the operating pipeline in that it can be collected during line down times. This also alleviates the requirement to time site visits with periods where there is sufficient variations in the pipeline operations to observe a wide range of process responses.

Two limitations in the current simulation model have been encountered. First, further refinement of the model for valve

dynamics should be undertaken. Step test data presented in Tables 2 and 3 are for the largest and smallest steps possible, showing significantly different results. Further testing at intermediate step sizes would provide a more complete picture. Secondly, the node mass balancing approach of the IDEAS simulator limits its ability to model multiple products in a pipeline. A simple procedure that accounts for the presence of multiple products was developed and demonstrated excellent results in calibrating the simulations against steady-state process data.

Enbridge is implementing the knowledge gained through dynamic simulation and intends to continue use of this highly effective engineering tool in assessing other control applications.

ACKNOWLEDGMENTS

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APPENDIX A

Figure A.1 illustrates two sections of the same pipeline, of cross-sectional area A , containing two product batches with different densities.

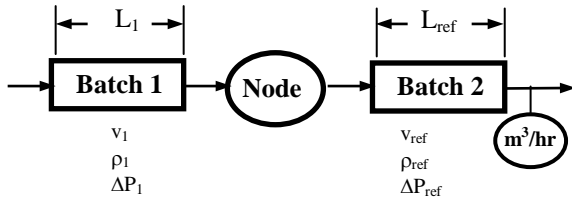


Figure A.1: Illustration of multiple products in the pipeline

Note that the flowrate is measured at the exit of the second section of pipeline. During calibration of the model, the flowrate at this point will be made to match the flowrate measured in the operating system. Hence the second section of pipeline is referred to as the reference section.

If it is required (e.g. by the simulation program) that the mass entering a point/node between the two batches is equal to the mass leaving the node then,

$$A \cdot v_1 \cdot \rho_1 = A \cdot v_{ref} \cdot \rho_{ref} \quad (A.1)$$

or,

$$\frac{v_1}{v_{ref}} = \frac{\rho_{ref}}{\rho_1} \quad (A.2)$$

However, in a closed pipeline filled with incompressible liquids, operating at constant volumetric flowrate, the mass flow at any point is not necessarily constant. Thus the mass balance expressed in Equation A.1 is not a good model for the actual pipeline because it does not allow for changes in mass (flow) along the pipeline resulting from materials with differing densities. These differences in mass flow along the pipeline will result in incorrect model predictions for flowrates, velocities and pressure drops.

Consider a pipe operating with fully developed turbulent flow, at a constant volumetric flowrate, with the inlet and outlet elevations the same. The pressure drop due to friction losses can be expressed for a single product of uniform material properties as [4]:

$$\frac{\Delta P}{\rho} = F \quad (A.3)$$

The Fanning Equation gives the following expression for friction losses in terms of the Friction Factor, f [4]:

$$F = \frac{2 \cdot v^2 \cdot L \cdot f}{D} \quad (A.4)$$

The predicted pressure drop in the IDEAS simulation model, which is based on mass balancing at nodes, will be correct for the fluid at the Reference point. The predicted pressure drop for other fluids in the pipeline, $\Delta P_1(v_1, L_1, f_1)$, will not be correct because the velocity and friction factor are incorrect. To compensate for this, the length of pipe could be adjusted to, say, L_1^* . With this, the 'correct' pressure drop, ΔP_1^* , could be written:

$$\Delta P_1^*(v_1, L_1, f_1) = \frac{2 \cdot \rho_1 \cdot v_1^2 \cdot L_1 \cdot f_1}{D} \quad (A.5)$$

ΔP_1^* could also be calculated in terms of the original pipe length with knowledge of the actual fluid velocity, v_{ref} :

$$\Delta P_1^*(v_{ref}, L_1, f_1^*) = \frac{2 \cdot \rho_1 \cdot v_{ref}^2 \cdot L_1 \cdot f_1^*}{D} \quad (A.6)$$

where f_1^* is the friction factor computed using v_{ref} .

Taking the ratio of the above two expressions yields a factor by which the original pipe length can be multiplied to correct for differences in material properties in batches:

$$\frac{\Delta P_1^*(v_1, L_1^*)}{\Delta P_1^*(v_{ref}, L_1)} = \frac{v_1^2 \cdot L_1^* \cdot f_1}{v_{ref}^2 \cdot L_1 \cdot f_1^*} = 1 \quad (A.7)$$

An approximate explicit expression for the friction factor is [4]:

$$f = 0.04 / N_{Re}^{0.16} \quad (A.8)$$

where $N_{Re} = \rho \cdot v \cdot D / \eta$. Utilizing this expression and Equation A.2, the following can be derived:

$$\frac{L_1^*}{L_1} = \left(\frac{\rho_1}{\rho_{ref}} \right)^{1.84} \quad (A.9)$$

APPENDIX B

Lambda Tuning Procedure

Lambda (λ) is the desired closed-loop time constant of an assumed linear first-order process. The following controller tunings are applied to achieve the desired response:

$$K_c = \frac{1}{PG} \left(\frac{1}{\lambda + DT} \right) \quad (\text{B.1})$$

$$I = TC \quad (\text{B.2})$$

where:

- K_c = Controller gain (% controller output / % change in error)
- I = Integral (seconds per repeat)
- λ = Closed-loop time constant (seconds)
- PG = Process Gain (% change in process variable / % change in controller output)
- TC = Time Constant (seconds)
- DT = Dead Time (seconds)

Lambda is chosen by the user so that:

$$\lambda \geq DT \text{ or } \lambda \geq TC$$

The open-loop process gain, time constant and dead time were determined by applying a 5% controller output step change into the simulator. The PG, TC and DT parameters were estimated by a least squares fit to the data generated. Tests were performed at minimum flow conditions and therefore represent system dynamics at this region of operation.

Output Linearization Procedure

The procedure used for determining linearization curves in this study is:

1. Normalized discharge pressure curves are first determined using the simulator by plotting normalized discharge pressure against controller output. Data is generated for 1, 2 and 3 operating pumps.
2. A smooth curve is fit to the data using either polynomial or power functions. The following power function was found to be convenient for fitting normalized pressure data as it is constrained to pass through the [0, 0] and [100, 100] points, and is easily inverted.

$$y = 100 \left\{ 1 - \left[1 - \left(\frac{x}{100} \right)^c \right]^a \right\} \quad (\text{B.3})$$

The two parameters 'a' and 'c' must be fit by nonlinear least squares.

3. The linearization curve is the inverse of the normalized discharge pressure curve. It can be determined in one of two ways:

- A polynomial can be fit to 1/(Normalized Discharge Pressure) values
- If a power function of the form [B.3] has been fit to the Normalized Discharge Pressure curve, the linearization curve is determined by Equation B.4:

$$y = 100 \left\{ 1 - \left[\left(1 - \frac{x}{100} \right)^{1/a} \right]^{1/c} \right\} \quad (\text{B.4})$$

4. Once linearization curves are determined, the gain curves for the modified process are redetermined. Lambda tunings are generated for this process using the maximum gain found over the entire operating region.